RZN c6_0

→ krátký test na konci bloku – teorie, z prvních dvou cvičení příklady

→ úkol – praktický na počítači, pro naučení se ve vlastním projektu, odevzdání 3 týdny cca před koncem semestru ??, neděle 3. ledna

→ Dk. dualita konjunkce a disjunkce

$\alpha \overset{\circ}{\vee} \beta = \neg_{\circ} (\neg_{\circ} \alpha \overset{\wedge}{\circ} \neg_{\circ} \beta)$ pro $\alpha \leq \beta$

a) st. ~~op~~ conj/disj + jakákoliv negace

$\alpha \wedge_S \beta = \min(\alpha, \beta)$
$\alpha \vee_S \beta = \max(\alpha, \beta)$

$\max(\alpha, \beta) = \neg_{\circ} (\underbrace{\min(\neg_{\circ}\alpha, \neg_{\circ}\beta)})$

z $\alpha \leq \beta$

$\beta = \neg_{\circ} \neg_{\circ} \beta$ · klesá $\neg_{\circ}$ (N1)

$\beta = \beta$ (N2)

$\beta \geq \alpha$ je symetrické s α)

b) Luk. op. + std. neg.

$\neg_S \alpha = 1 - \alpha$
$\alpha \wedge_L \beta = \max(\alpha+\beta-1, 0)$
$\alpha \vee_L \beta = \min(\alpha+\beta, 1)$

$\min(\alpha+\beta, 1) = 1 - \max(1-\alpha + 1-\beta - 1, 0)$

$\min(\alpha+\beta, 1) = 1 - \max(1-\alpha-\beta, 0)$

1) $\beta + \alpha \leq 1$

$\beta + \alpha = 1 - (1 - \alpha - \beta)$

$\beta + \alpha = \alpha + \beta$

2) $\beta + \alpha \geq 1$

$1 = 1 - 0$

2) → pří.

$$(\alpha \overset{\circ}{\vee} \beta) \wedge_L (\alpha \overset{\circ}{\vee}_S \neg \beta) = \alpha$$

$\alpha \wedge_P \beta = \alpha \cdot \beta$

$\alpha \overset{\circ}{\vee} \beta = \alpha + \beta - \alpha\beta$

0. pro Boolovskou logiku
1. st. operace
2. produktové op.
3. Luk. op

0. $(\alpha \vee \beta) \wedge_L (\alpha \vee_S \neg \beta) = \alpha$

$\max((\alpha \vee \beta) + (\alpha \vee (1-\beta)) - 1, 0) = \alpha$

$\max(\alpha + \beta + \alpha + 1 - \beta - 1, 0) = \alpha$

$\max(2(\alpha), 0) = \alpha$

1. $(\max(\alpha, \beta)) \wedge_L (\max(\alpha, 1-\beta)) = \alpha$

$\max(\max(\alpha, \beta) + \max(\alpha, 1-\beta) - 1, 0) = \alpha$

a)

→ nalezení protipříkladu

$\alpha = 0{,}7$ a $\beta = 0{,}5$, pak

se výraz $\neq \alpha$

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2. $(\alpha+\beta-\alpha\beta) \underset{L}{\wedge} (\alpha+(1-\beta)-\alpha(1-\beta)) = \alpha$

$\max(\alpha \cancel{+\beta} - \cancel{\alpha\beta} \cancel{+\alpha} + \cancel{(1-\beta)} - \cancel{\alpha} + \cancel{\alpha\beta} \cancel{-1}, 0) = \alpha$

$\max(\alpha, 0) = \alpha$

3. $(\min(\alpha+\beta, 1)) \underset{L}{\wedge} (\min(\alpha+1-\beta, 1)) = \alpha$

$\max(\min(\alpha+\beta, 1) + \min(\alpha+1-\beta, 1) - 1, 0) = \alpha$

$\alpha = 0{,}3 \quad \beta = 0{,}2$ je protipříklad

$\neq \alpha$ platí

10) → př.

$\underset{SA}{\neg}\alpha = \dfrac{1-\alpha}{1+\lambda\alpha} \quad \lambda \in (-1, \infty)$ → platí, že je plnou fuzzy negací

↘ důkaz přes vlastnosti negace, její definici, zda pro $\underset{SA}{\neg}\alpha$ tyto platí

$$\underset{SA}{\neg}\underset{SA}{\neg}\alpha = \frac{1-\frac{1-\alpha}{1+\lambda\alpha}}{1+\lambda\frac{1-\alpha}{1+\lambda\alpha}} = \frac{\frac{1+\lambda\alpha-1+\alpha}{1+\lambda\alpha}}{\frac{1+\lambda\alpha+\lambda-\lambda\alpha}{1+\lambda\alpha}} =$$

$$= \frac{\lambda\alpha+\alpha}{1+\lambda} = \alpha\frac{1+\lambda}{1+\lambda} = \alpha$$

$\alpha \leq \beta$

$\underset{SA}{\neg}\alpha = \dfrac{1-\alpha}{1+\lambda\alpha} \quad \underset{SA}{\neg}\beta = \dfrac{1-\beta}{1+\lambda\beta}$

$1+\lambda\alpha \leq 1+\lambda\beta \wedge 1-\alpha \geq 1-\beta$

$\Downarrow$

$\underset{SA}{\neg}\alpha \geq \underset{SA}{\neg}\beta$

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Ověřte $\alpha \underset{O}{\wedge} (\alpha \overset{R}{\underset{O}{\Rightarrow}} \beta) = \alpha \underset{S}{\wedge} \beta$

a) Luk b) std. c) alg.

$\alpha \underset{S}{\wedge} \beta = \min(\alpha, \beta)$

$\alpha \underset{L}{\wedge} \beta = \max(\alpha+\beta-1, 0)$

$\alpha \overset{R}{\underset{S}{\Rightarrow}} \beta = \begin{cases} 1 & \alpha \le \beta \\ \beta & \text{jinak} \end{cases}$

$\alpha \overset{R}{\underset{L}{\Rightarrow}} \beta = \begin{cases} 1 & \alpha \le \beta \\ 1-\alpha+\beta & \text{jinak} \end{cases}$

$\alpha \underset{A}{\wedge} \beta = \alpha\beta$

$\alpha \overset{R}{\underset{A}{\Rightarrow}} \beta \begin{cases} 1 & \alpha \le \beta \\ \frac{\beta}{\alpha} & \text{jinak} \end{cases}$

b)

$\min(\alpha, \alpha \overset{R}{\underset{S}{\Rightarrow}} \beta) = \min(\alpha, \beta)$

$\alpha \le \beta$: $\alpha = \alpha$

$\alpha > \beta$: $\beta = \beta$ $(\alpha \overset{R}{\underset{S}{\Rightarrow}} \beta = \beta)$

platí

a)

$\max(\alpha + (\alpha \overset{R}{\underset{L}{\Rightarrow}} \beta) - 1, 0) = \min(\alpha, \beta)$

$\alpha \le \beta$: $\max(\alpha + 1 - 1, 0) = \alpha$

$\alpha > \beta$: $\max(\alpha + (1-\alpha+\beta) - 1, 0) = \beta$

$\beta = \beta$

platí

c) $\alpha(\alpha \overset{R}{\underset{A}{\Rightarrow}} \beta) = \min(\alpha, \beta)$

platí

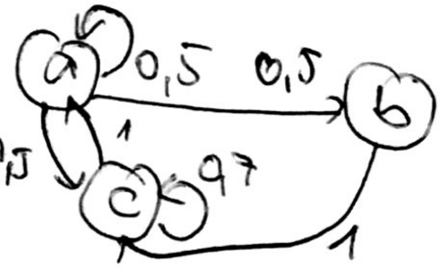
$\alpha \le \beta$: $\alpha(1) = \alpha$

$\alpha > \beta$: $\alpha\left(\frac{\beta}{\alpha}\right) = \beta$

$\beta = \beta$

$$R \quad (x,y) \qquad \begin{array}{c|ccc} & a & b & c \\ \hline a & .1 & .2 & .3 \\ b & .4 & .5 & .6 \\ c & .7 & .8 & .9 \end{array}$$

$$S \qquad \begin{array}{c|ccc} & a & b & c \\ \hline a & .5 & .5 & .5 \\ b & 0 & 0 & 1.0 \\ c & 1.0 & 0 & .7 \end{array}$$



$$R \underset{S}{\circ} S = \begin{array}{|c|c|c|} \hline 0,5 & 0,1 & 0,3 \\ \hline 0,6 & 0,4 & 0,6 \\ \hline 0,9 & 0,5 & 0,8 \\ \hline \end{array}$$

~~0,1 0,3~~ 0,7

0,5

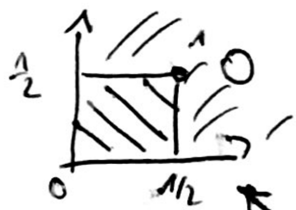
$'R \circ S'(a,c)$

$$\left[R(a,a) \underset{S}{\wedge} S(a,c)\right] \overset{S}{\vee} \left[R(a,b) \underset{S}{\wedge} S(b,c)\right] \overset{S}{\vee} \left[R(a,c) \underset{S}{\wedge} S(c,c)\right] = \max\begin{pmatrix} \min(0,1\ ;\ 0,5) \\ \min(0,2\ ;\ 1,0) \\ \min(0,3\ ;\ 0,7) \end{pmatrix} =$$

$$= \max(0,1;\ 0,2;\ 0,3)$$

obdoba násobení matic

$$R(x,y) = \begin{cases} x+y & x,y \in [0, \tfrac{1}{2}] \\ 0 \end{cases}$$



$$S(y,z) = \begin{cases} \cancel{x \cdot z} & x \in [0,1] \\ 0 & \text{jinak} \end{cases}$$

$$R \underset{A}{\circ} S = \max_{y \in [0, \frac{1}{2}]} (x+y)(\cancel{y}\,\cancel{z}) =$$

derivace podle y

$$= \left(x + \frac{1}{2}\right)\frac{1}{2} \cdot z = \frac{xz}{2} + \frac{z}{4}$$

pro $x,y \in [0, \frac{1}{2}]$

	a	b	c
a	1	0,3	y
b	0	1	0,5
c	x	0	1

R

a) R – ~~je~~ částečně uspořádání → dopočtení prvtů

→ reflexivita

$E \leq R$

1	0	0
0	1	0
0	0	1

E

→ antisymetrie

$R(a,b) \wedge_s R(b,a) \leq E(a,b)$

$\min(-\|-) \leq 0$

→ aspoň jeden z prvtů je 0

→ tranzitivita

RoR

1	0,3	
	1	0,5
x		1

← ~~max~~ $(y, 0{,}3, y)$

$0 = \min(0, 0, x)$

$\min(0{,}3; x) = 0$

~~max~~ 0

$R \geq RoR$

→ $X = 0$

$\max(x; 0{,}3) \leq y$

$\boxed{y \geq 0{,}3}$

b) A – uspořádání ~~[illegible]~~

	a	b	c
a	1	0,3	x
b	0	1	0,5
c	y	0	1

→ $y = 0$

RoR

1	0,3	
0,5y	1	0,5
y	0,3y	1

→ $\max(x; 0{,}15)$

$x \geq \max(y; 0{,}15)$

$\boxed{x \geq 0{,}15}$

c) L-uspořádání

c)

1	0,3	x
z	1	0,5
y	w	1

$z \leq 0,7$
$w \leq 0,5$
$\vdots$
$\rightarrow z$
antisymetrie

$\langle John : \exists child.Man \mid 0{,}9 \rangle$

$\langle \exists child.Man \sqsubseteq Father \mid 0{,}8 \rangle$

$glb(\mathcal{K}, \langle John : Father \rangle)$

dokázat, že John je opravdu otec

John

$\langle \forall child.\neg Man \mid 1 - x_3 \rangle$ $\langle \exists child.Man \mid 0{,}9 \rangle$

$\langle Father \mid x_4 \rangle$ $\langle child \mid x_1 \rangle$

$\langle Man, x_2 \rangle$

w $\langle \neg Man, x_5 \rangle$

$x_{w:Man} \leq 1 - x_5$

$(1 - x_3) + x_1 - 1 \leq x_5 \leq y \leq (1 - x_3) + x_1$

$x_{John:Father} \geq x_4$

$\cancel{x_1 \leq 1 - 0{,}9 = 0{,}1}$

$\cancel{x_1 \leq 1 - x_1}$

$\cancel{x_2 \leq 1 - x_2}$

$\boxed{x_1 + x_2 = 0{,}9 + 1 - \cancel{1}}$ $\rightarrow$ $x_1 + x_2 = 1{,}9$

$x_3 \leq x_4 + 1 - 0{,}8$

$x_{w:man} \geq x_2$

$x_{(John,w):child} \geq x_1$

$nnf(\neg \exists child.Man) = \forall child.nnf(\neg Man) =$
$= \forall child.\neg Man$

$\Rightarrow$ je opravdu otec $\rightarrow$ minimalizace

$x_{John:Father} \geq x_4$

$\downarrow$

$x_3 \uparrow, x_1 \downarrow, x_5 \uparrow, y \uparrow, x_4 \downarrow$

proto $x_{w:man} \downarrow, x_2 \downarrow$