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What Power Makes them Moving

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1. Particle systems
2. Rigid body dynamics
3. Motion balance of bi-peds

Particle Systems

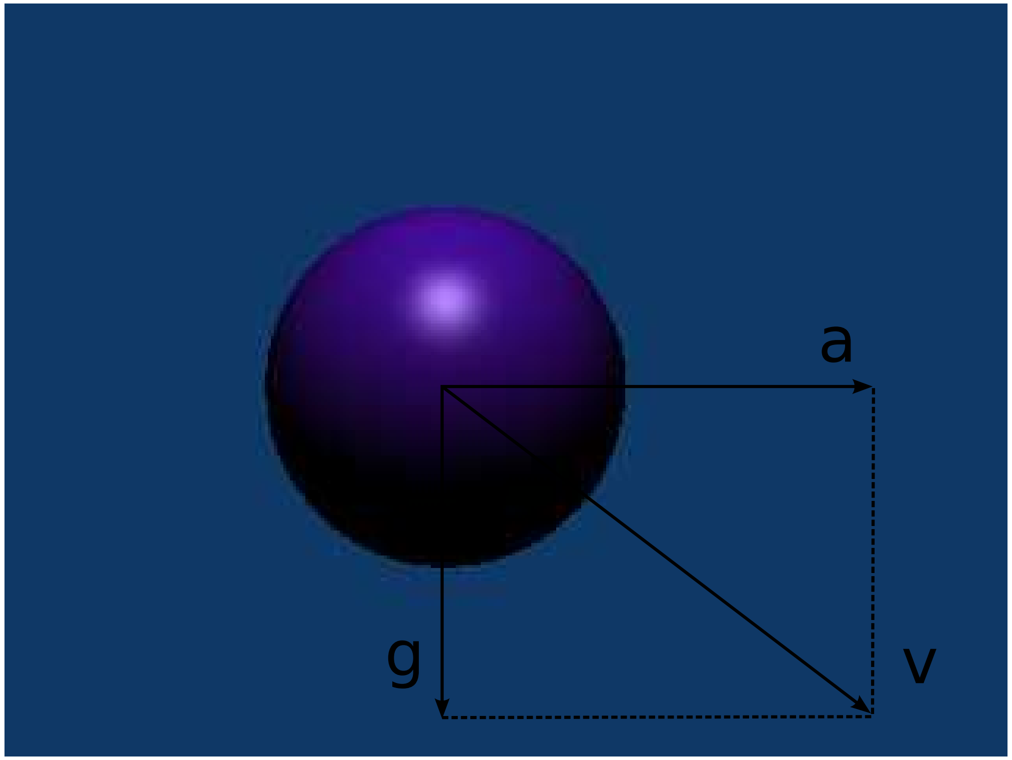
- using 3DOF or 6DOF elements instead of classical polygonal objects
- representation of "fuzzy" objects (fog, smoke, clouds, water)
- PS represent volume $\times$ polygons represent surface (border representation)
- PS is dynamic entity $\Rightarrow$ develops during its life
- object represented by PS has no predetermined shape $\rightarrow$ it is given by simulation

Advantages & Disadvantages

+ a particle is simple $\Rightarrow$ simple manipulation
+ PS is given by procedural definition $\Rightarrow$ simpler realization of complex objects,
+ $\Rightarrow$ possibility of automatic LOD
+ simpler representation of dynamic behavior

\- complex objects $\Rightarrow$ high computation cost

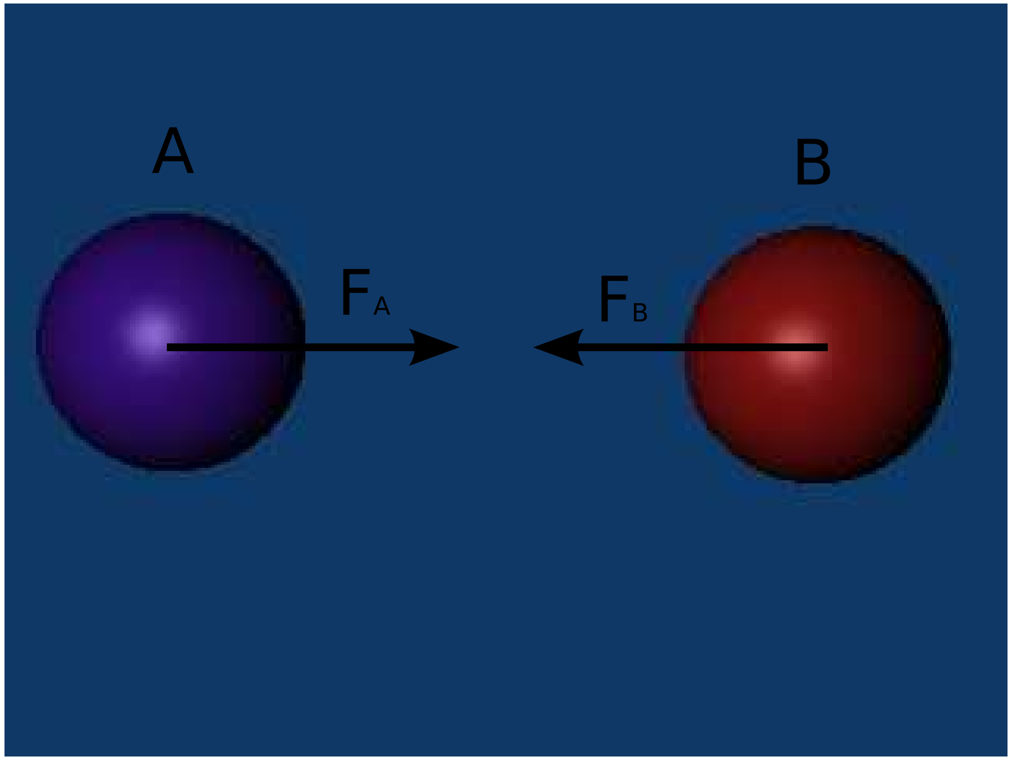
Where the particle will go at the nearest time?



- the second Newton's law acts to everybody: $\mathbf{F} = m\frac{d\mathbf{v}}{dt} = m\frac{d^2\mathbf{r}}{dt^2}$
- we want to compute position at any time from the given force:

$$\mathbf{r}(t) = \frac{1}{m}\int_{t_0}^{t}\int_{t_0}^{t}\mathbf{F}(t)dtdt$$

What two particles think about each other?



- Newton's gravitation law: $F = \kappa \frac{M_1 M_2}{r^2}$
- This law acts between each two physical objects.
- The gravitational force belongs to weak forces in nature.

Which forces act on the system?

1. unary forces
   gravity, resistant forces of the particle environment
2. N-ary forces
   forces acting to fixed set of particles
3. spacial interaction forces
   attraction and repulsion forces, *oriented particles*

Unary Forces

- gravitation force (mass)

$$\mathbf{G} = m\mathbf{g}$$

  gravitational acceleration $\mathbf{g} = (0, -g, 0)$

- resistant forces (viscous friction)

$$\mathbf{F} = -k_d\mathbf{v}$$

  k_d is *coefficient of drag*

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N-ary Forces

- forces acting on pairs of particles, modeled by system of sprigs:

$$\mathbf{F}_a = -\mathbf{F}_b = -\left[k_s(|\mathbf{l}| - r) + k_d \frac{\mathbf{i}.\mathbf{l}}{|\mathbf{l}|}\right] . \frac{\mathbf{l}}{|\mathbf{l}|} \begin{cases} \mathbf{l} & = & \mathbf{a} - \mathbf{b} \\ \mathbf{i} & = & \mathbf{v}_a - \mathbf{v}_b \end{cases}$$

- k_s is spring constant
- k_d is dumping constant
- r is rest length
- $\mathbf{a}$ and $\mathbf{b}$ are positions of particles

Spacial Interaction Forces

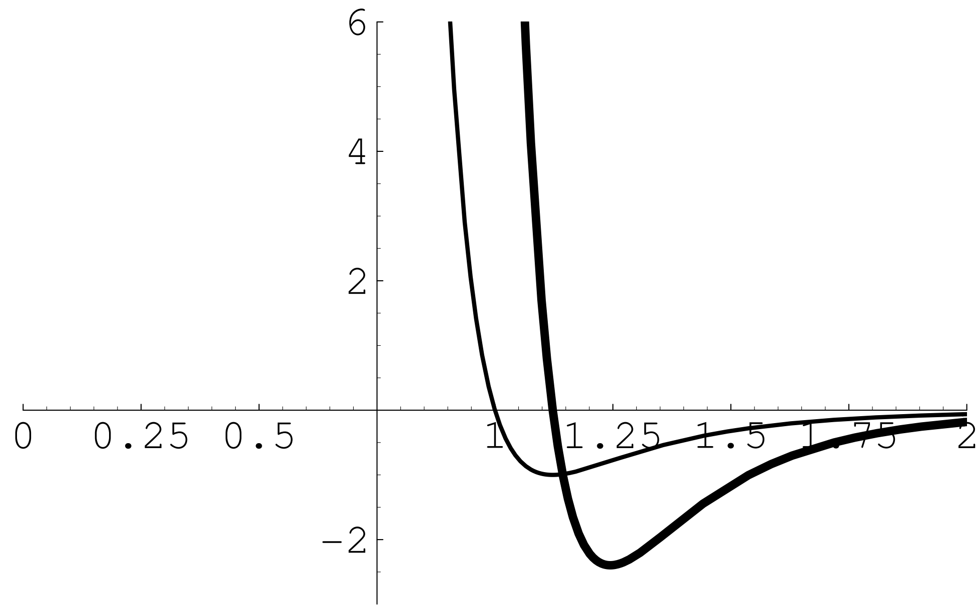
- forces acting on pairs or n-tuplets of particles
- used to simulate fluids, and large scale particle models
- force calculation $O(n^2) \Rightarrow$ disadvantage for large-scale spatial interaction
- potencial fields - *Lennard-Jones potential function*

$$\phi_{LJ}(r_{ij}) = A|r_{ij}|^{-n} - B|r_{ij}|^{-m}$$

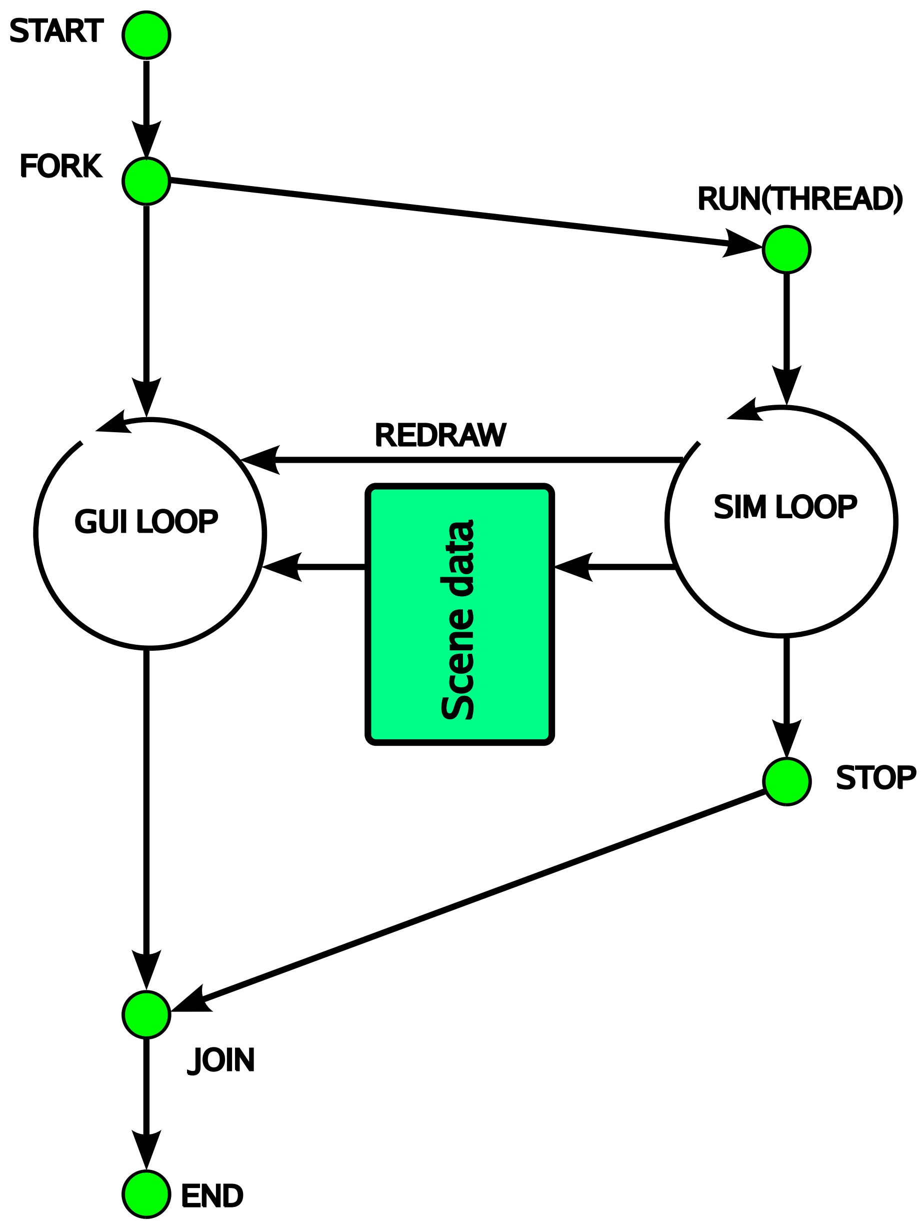
$$\mathbf{F}_{ij} = -\nabla_r \phi_{LJ}(|\mathbf{r}_{ij}|)$$

Example:

$$\phi_{LJ}(r_{ij}) = 4\varepsilon\left(\left(\frac{\sigma}{r}\right)^{6} - \left(\frac{\sigma}{r}\right)^{12}\right)$$



How the Machine Works?



The simplest model can use shared memory ⇒ threaded architecture.

More complicated model represents distributed world ⇒ problem of synchronization

How They Can be Visualized?

- Light emitting method
- Polygonal object substitution
- Volume object substitution
- Implicit surface rendering

Light emitting method

- particle is displayed in therms of a point light source
- the particle shape, size and distance from an observer determines covered pixels
- no shadows, no depth-sorting is needed
- explosions, fire

Polygonal object substitution

- each particle is replaced by a polygonal object
- the object can vary in complexity from one polygon to whole mash
- flocks, herds

Volume object substitution

- similar to the previous approach
- a volume object substitutes the particles
- approximate comets, sparks, water droplets

Implicit Surface Rendering

- particles are replaced by sources of a potential field $\Rightarrow$ *blobby objects*
- the implicit surface of an object is defined by superposition of all potential sources:

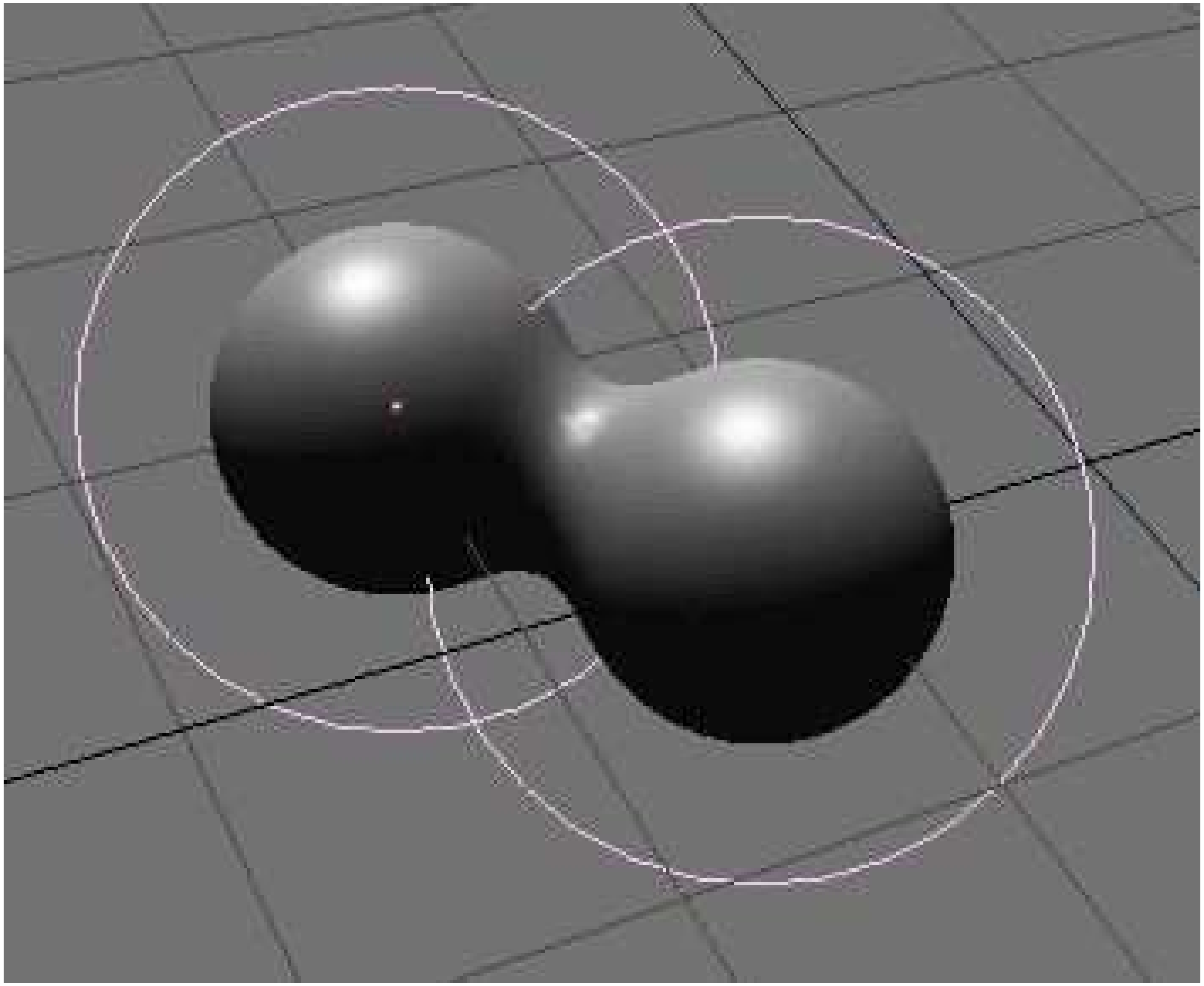
$$D_{blob}(r) = \sum_{i=0}^{n} B_i e^{-A_i r_i^2} = T,$$

where T is te equipotential which should be rendered, A and B are constants and r_i is a generalized distance to the potential source p_i:

$$r_i = \frac{|\mathbf{p} - \mathbf{p_i}|}{R_i},$$

where R_i is the threshold of distance where the potential value falls to zero.

Implicit Surface Rendering



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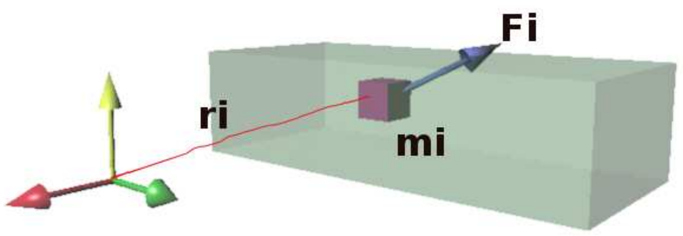
More Particles Together??



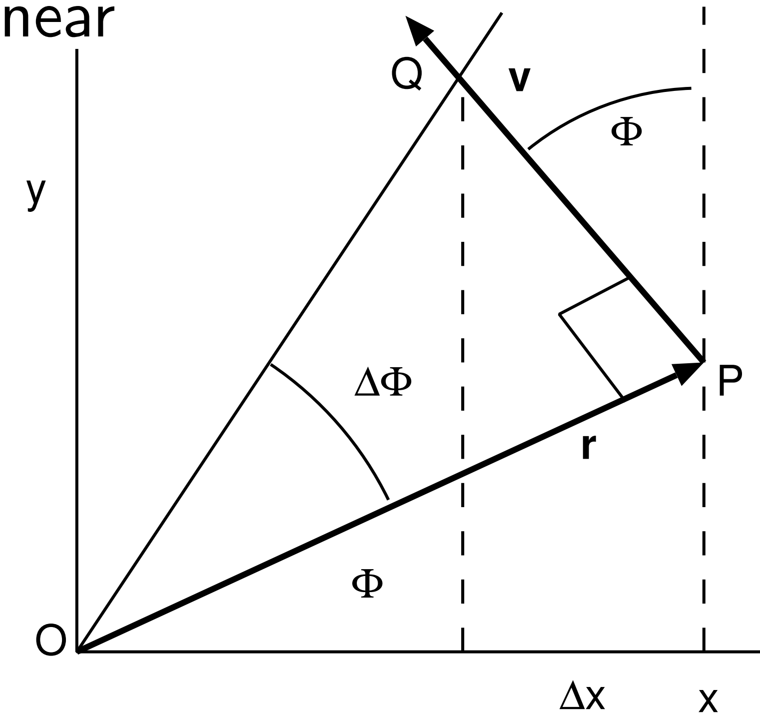
- a small part of a solid with constant mass $\mathbf{F}_i = m_i \frac{d^2\mathbf{r}_i}{dt^2} = \frac{d^2(m_i\mathbf{r}_i)}{dt^2}$
- the total force to the solid $\mathbf{F} = \sum_i \mathbf{F}_i = \frac{d^2 \sum_i (m_i\mathbf{r}_i)}{dt^2}$
- let's write $\mathbf{R} = \frac{\sum_i m_i\mathbf{r}_i}{M}$ where $M = \sum_i m_i$ then $\mathbf{F} = M\frac{d^2\mathbf{R}}{dt^2}$ (analogy to $\mathbf{F} = m\mathbf{a}$)

...and what is $\mathbf{R}$?

Let's rotate the solid in 2D

- there is an analogy between angular and linear velocity: $\omega = \frac{d\varphi}{dt} \sim \mathbf{v} = \frac{d\mathbf{r}}{dt}$

$$\begin{aligned} \Delta x &= -\mathbf{PQ}\sin\varphi = -r\Delta\varphi\tfrac{y}{r} \\ &= -y\Delta\varphi \\ \Delta y &= x\Delta\varphi \end{aligned}$$

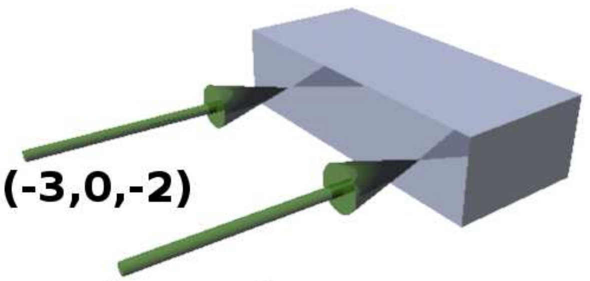


- the force performs the work: $\Delta W = F_x\Delta x + F_y\Delta y = \underbrace{(xF_y - yF_x)}_{?}\Delta\varphi$

- the force making the solid to rotate → torque: $N_i = x_iF_{yi} - y_iF_{xi}$, $N = \sum_i N_i, \mathbf{N} = \mathbf{r} \times \mathbf{F}$

- if all forces acting to the solid are in equilibrium then total force is zero, total torque is zero and the solid is in equilibrium: $\Delta W = N\Delta\varphi = 0$

Example 1



- External forces $F = (0, 0, f)$ act at points $x(t) + (-3, 0, -2)$ and $x(t) + (3, 0, -2)$
- the total force $\mathbf{F} = (0, 0, 2f)$ and $a = \frac{2f}{M}$
- the torgue due to the first force: $((x(t) + (-3, 0, -2)) - x(t)) \times \mathbf{F} = (-3, 0, -2) \times \mathbf{F}$, analogically the torque due to the second force
- the total torque $\tau = (0, 0, -2) \times (0, 0, f) = \mathbf{0}$

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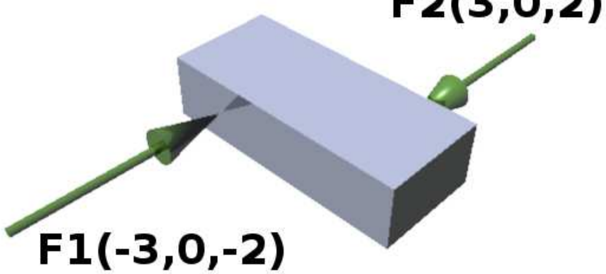
Example 2



- External forces $F_1 = (0, 0, f)$ and $F_2 = (0, 0, -f)$ act at points $x(t) + (-3, 0, -2)$ and $x(t) + (3, 0, 2)$
- the total force $\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = \mathbf{0}$
- the torgue due to the first force: $((x(t) + (-3, 0, -2)) - x(t)) \times \mathbf{F_1} = (-3, 0, -2) \times \mathbf{F_1}$, analogically the torque due to the second force
- the total torque $\tau = (0, 3f, 0) + (0, 3f, 0) = (0, 6f, 0)$

Angular Momentum (Moment of momentum)

- torque expressed using acceleration:

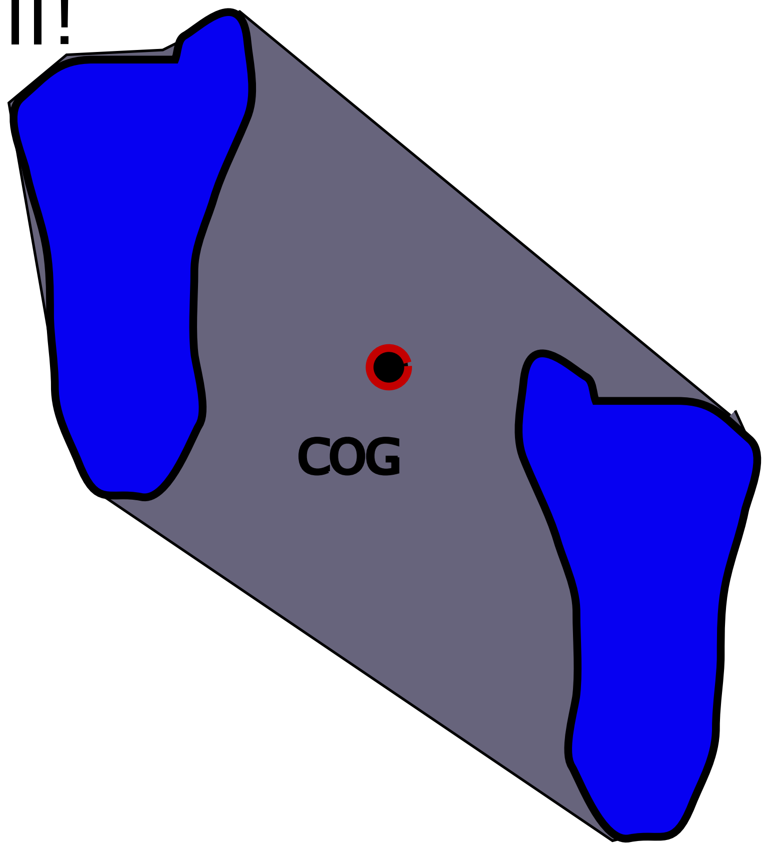
$$N = xF_y - yF_x = xm\frac{d^2y}{dt^2} - ym\frac{d^2x}{dt^2} = \frac{d}{dt}\left(\underbrace{xm\frac{dy}{dt} - ym\frac{dx}{dt}}_{?}\right)$$

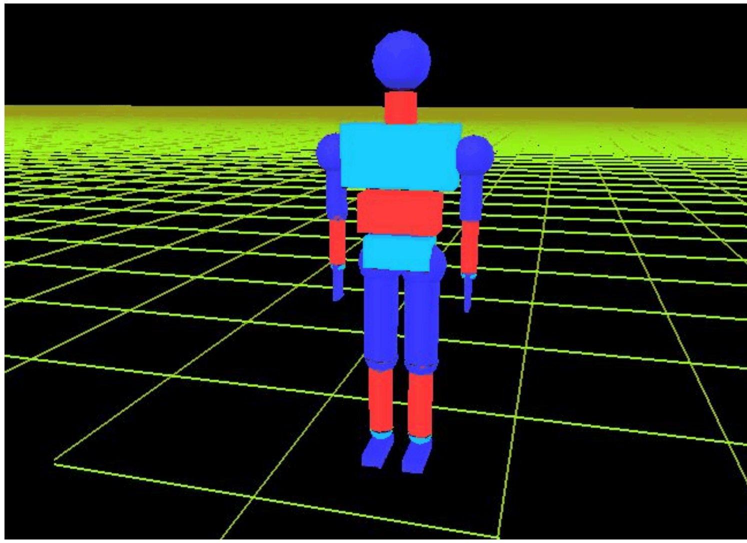
- the angular momentum: $L = xm\frac{dy}{dt} - ym\frac{dx}{dt}$
- the rate of change of angular momentum according to arbitrary axis equals to the total torque of external forces according to the same axis: $N = \sum_i N_i = N_{ext} = \frac{dL}{dt}$
- the part of a solid has angular momentum: $L_i = m_i v_i r_i = m_i r_i^2 \omega$
- the total angular momentum $L = I\omega$, where $I = \sum_i m_i r_i^2 \rightarrow$ moment of inertia

Moment of Inertia

- for solid object $I = \sum_i m_i(x_i^2 + y_i^2) = \int_i (x^2 + y^2) dm$
- the total moment of inetria for a solid is $I = \sum_i I_i$ according to the axis of rotation
- moment of inetria according to an arbitrary axis $I = I_T + MR_T^2$
- ...but generally I is a matrix: $\begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{pmatrix}$ $\rightarrow$ *inertia tensor*
- then $L_i = \sum_j I_{ij}\omega_j$
- motion equation for rotation is: $\mathbf{N} = I\frac{d^2\varphi}{dt^2}, \varphi = (\varphi_x, \varphi_y, \varphi_z)$

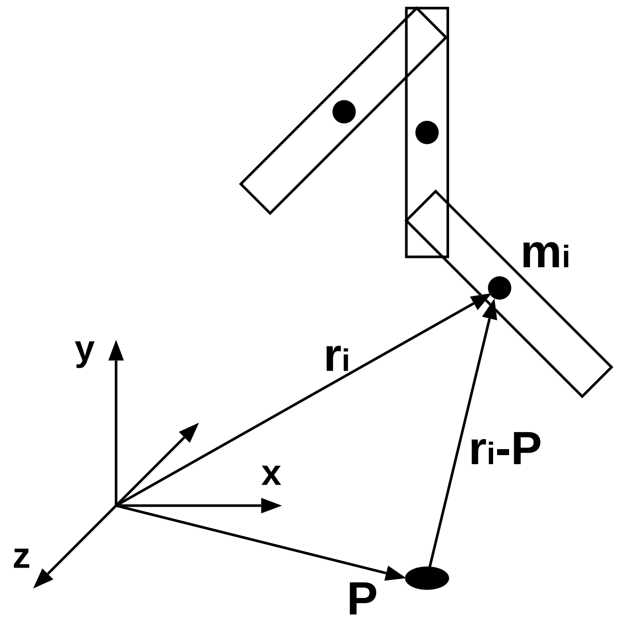
Do not fall!





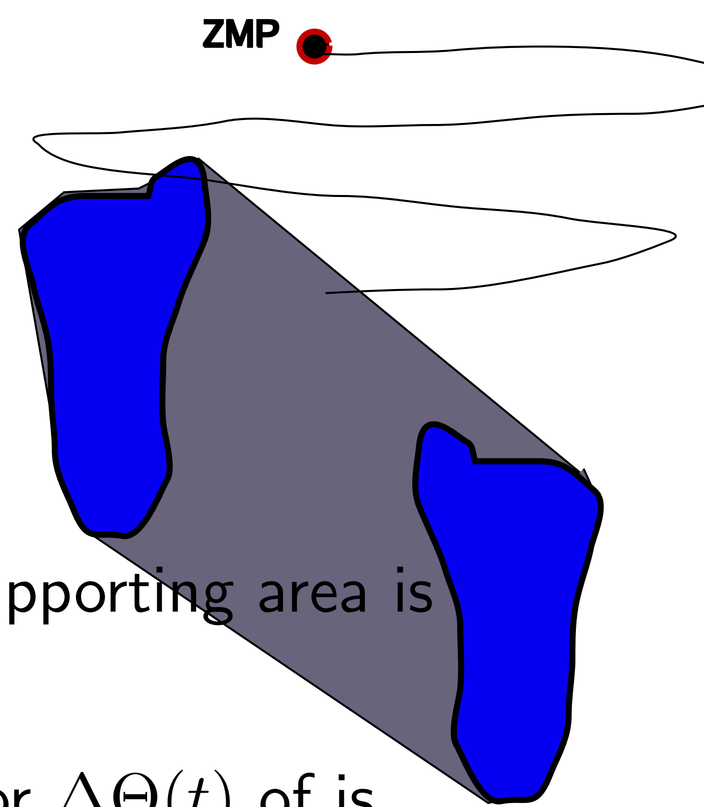
- for static bi-ped model the *center of gravity* projected to the ground must lie in so called *supporting area*
- for moving two legged model so called *zero moment point* ZMP must be computed

Zero Moment Point



- according to the law of conservation of moment of momentum: $\frac{d\mathbf{L}}{dt} = \sum_i \mathbf{N}_i$
- for the walking bi-ped both sides of the equation must be zero
- $\sum_i^n [\underbrace{(\mathbf{r}_i - P)}_{force\ arm} \times \underbrace{m_i(-\ddot{\mathbf{r}} + \mathbf{g})}_{force}] = 0$
- solving the equation we will get solution in form $\mathbf{P} = (P_x, 0, P_z)$

Swing, swing, swing...



- then the test whether the ZMP lies in the supporting area is evaluated
- if not then the correction configuration vector $\Delta\Theta(t)$ of is computed...
- ...and added to the current configuration: $\Theta(t+\Delta t) = \Theta_0(t) + \Delta\Theta(t)$ [Ko and Badler(1996)]
- computing $\Delta\Theta(t)$ is the optimization task with goal to get mimimal correction necessary to get ZMP lying in the supporting area [Tak et al.(2000)]

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