1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26

Cloth Modeling and Animation

Roman Berka
http://vyuka.iim.cz/a4m39mma:a4m39mma

1. History and classification
2. Modeling method overview
3. Representing fabrics particle system
4. Physically-Based Modelling

Virtual garment design and simulation

- subject of research since 1986 – Weil
- interesting for both industry and computer graphics
  - industry – study of mechanical properties
  - computer graphics – visualization, real-time animation

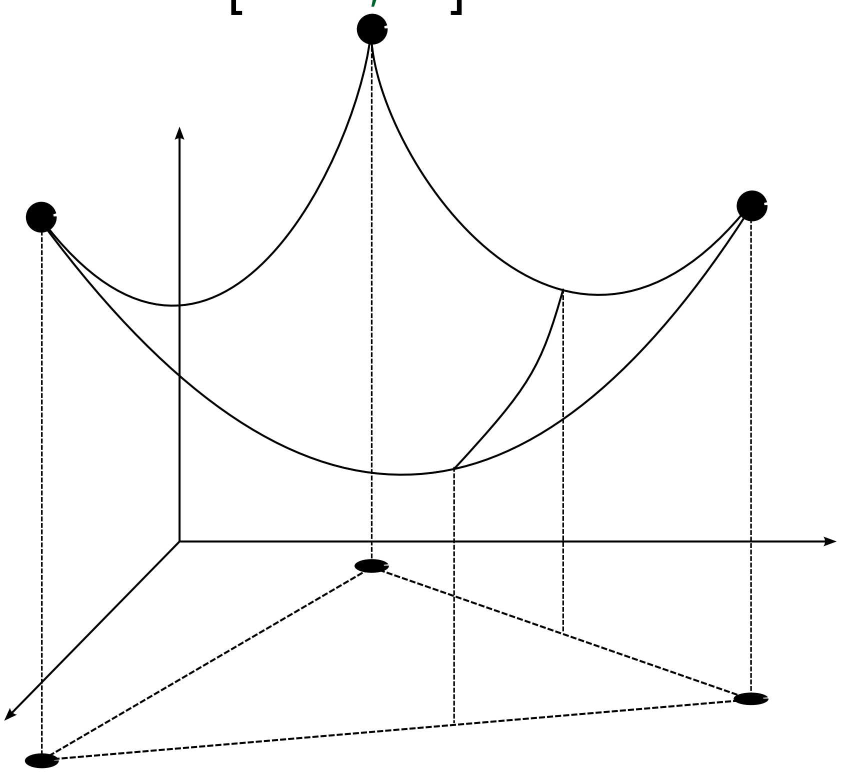
Classifications of models

- internal consistency → discrete vs. continuous models
- physical nature → geometric vs. physically based models

Try catenary curve

- approximation of folds on piece of square cloth [Weil,86]
- representation with catenary curves:

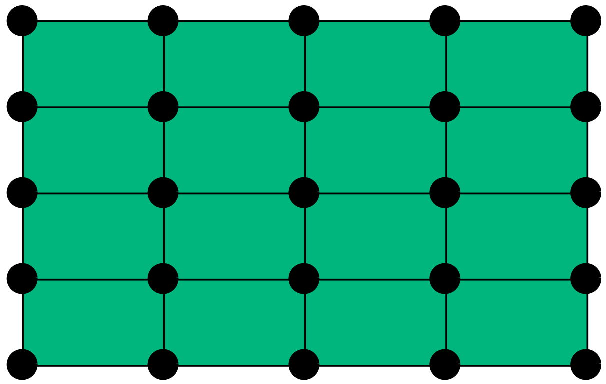
$$
\begin{aligned}
x(t) &= t \\
y(t) &= \frac{a}{2}\left(e^{\frac{t}{a}} + e^{-\frac{t}{a}}\right) = a\cosh\left(\frac{t}{a}\right)
\end{aligned}
$$



- the construction has two phases:

1. adding new constraint points until desired resolution is achieved
2. refinement using relaxation method to enforce distance constraints

Discrete Mesh Representation



- the general discrete model is based on grid of particles (mass) nodes
- forces on each node is computed based on its position and velocity and positions and velocities of topology neighbor nodes

Discrete Mesh Representation

- for the force F acting on each node the trajectory of each particle with mass m_i at position x_i is computed as

$$F(\mathbf{x}, \mathbf{v}) = m_i . \frac{d^2\mathbf{x}_i}{dt^2},$$

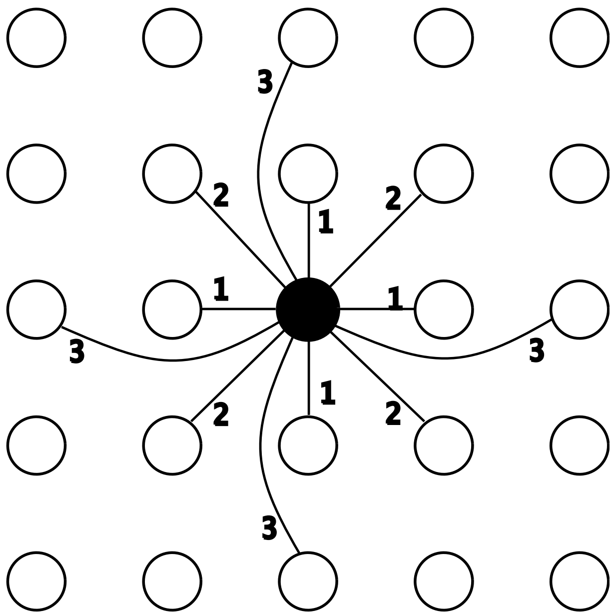
  where $\mathbf{x}$ is vector of positions and $\mathbf{v}$ is vector of velocities of all particles

- the system of ordinary differential equations is then solved

Mass-Spring Models

- the model based on mass-nodes
- connected with 3 kinds of springs [Provot, 95]

1. structural springs
2. shear springs
3. flexion springs



Elastic forces

- forces between two nodes at positions $\mathbf{x}_i$ and $\mathbf{x}_j$:

$$F^e_{ij} = k_{ij}(|\mathbf{x}_i - \mathbf{x}_j| - l_{ij})\frac{\mathbf{x}_i - \mathbf{x}_j}{|\mathbf{x}_i - \mathbf{x}_j|},$$

  where k_{ij} is the elastic modulus of the spring and l_{ij} is the rest length of the spring

- k_{ij} - large values for structural springs, small values for shear and flection springs

Dumping Forces

- viscous forces represent energy dissipation due to internal friction
- the viscous forces are modeled as

$$F_{ij}^d = d_{ij}(\mathbf{v}_i - \mathbf{v}_j),$$

where d_{ij} is the dumping factor

- unnatural beavior
- the projection of damping term into spring is used:

$$F_{ij}^d = d_{ij}\frac{(\mathbf{v}_i - \mathbf{v}_j)(\mathbf{x}_i - \mathbf{x}_j)}{|\mathbf{x}_i - \mathbf{x}_j|^2}(\mathbf{x}_i - \mathbf{x}_j)$$

- the resulting force is plugged into the motion equation

Continuous Representation

- the idea of cloth as a continuous flexible matherial
- Feynman,1986 idea of membrane – strain and bend energy functions:

$$E_s = \frac{E}{1-\nu^2}(u_{xx}^2 - u_{yy}^2) + \frac{2\nu E}{1-\nu^2} u_{xx} u_{yy},$$

  E is Young's modulus, ν Poisson's ratio and u_{ii} is the strain.
- the energy of bending:

$$E_b(S) = \int_0^{v_{max}} \int_0^{u_{max}} c_1 \kappa^2 du dv,$$

  κ is the principal curvature of the surface, c_1 is mechanical stiffness parameter.
- based on distace between points and simple measure of curvature
- does not solve interactions with environment

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26

Continuum Mechanics

- the dynamics of elastic model Terzopoulos, 1987
- Lagrange's equation of motion:

$$\frac{\partial}{\partial t}\left(\mu\frac{\partial \mathbf{r}}{\partial t}\right) + \gamma\frac{\partial \mathbf{r}}{\partial t} + \frac{\delta\varepsilon(\mathbf{r})}{\delta \mathbf{r}} = f(\mathbf{r}, t),$$

$\mathbf{r}(a,t)$ is position of point a in time t, $\mu(a)$ is the mass density of the cloth at a, $\gamma(a)$ is the dumping factor at a, $f(\mathbf{r},t)$ represents externally applied forces and $\varepsilon(\mathbf{r})$ represents potential energy of the deformation

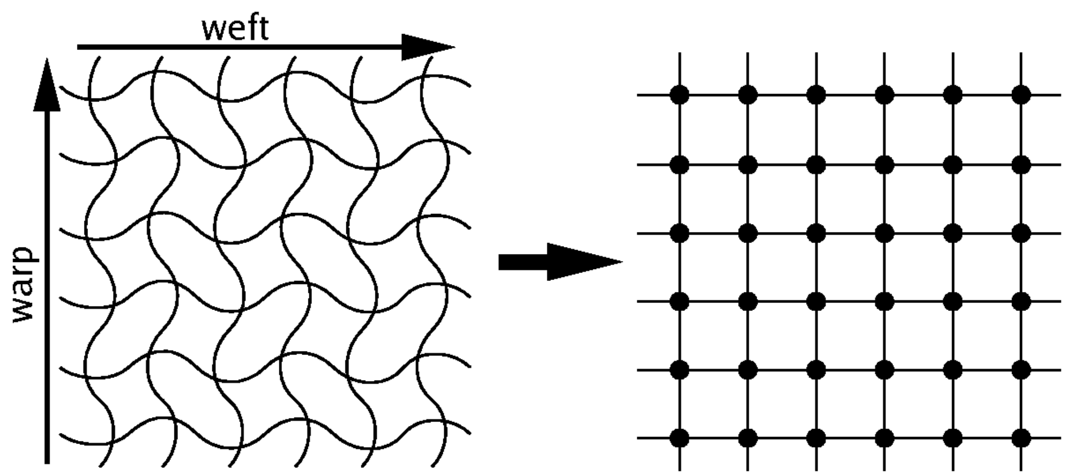
- the first term – inertial force, the second term – dumping, the third term – elastic force due to deformation
- adopted by Magnenat-Thalmann and Thalmann for development of elastic deformation model

MIRALab - University of Geneva

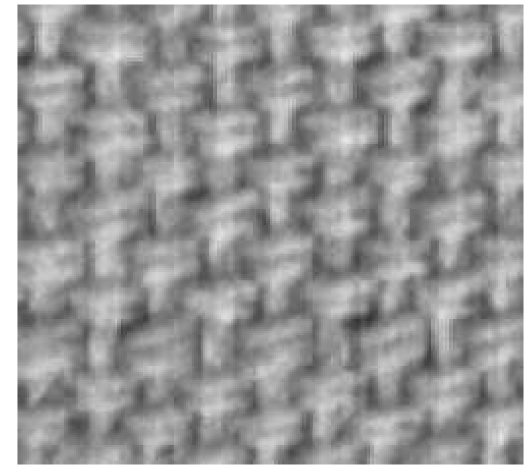
One of the top world research centers in the area of cloth animation.

Cloth is a mechanism, not a continuous material

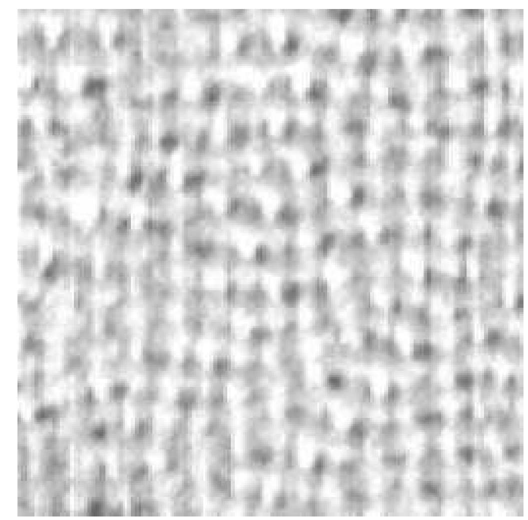
- woven fabrics structure ↓ *warp* and → *weft*
- representin of yarn crossings by particles:
  - compression and stretching
  - bending
  - trellising
  - twisting



a) plain weave b) particle representation

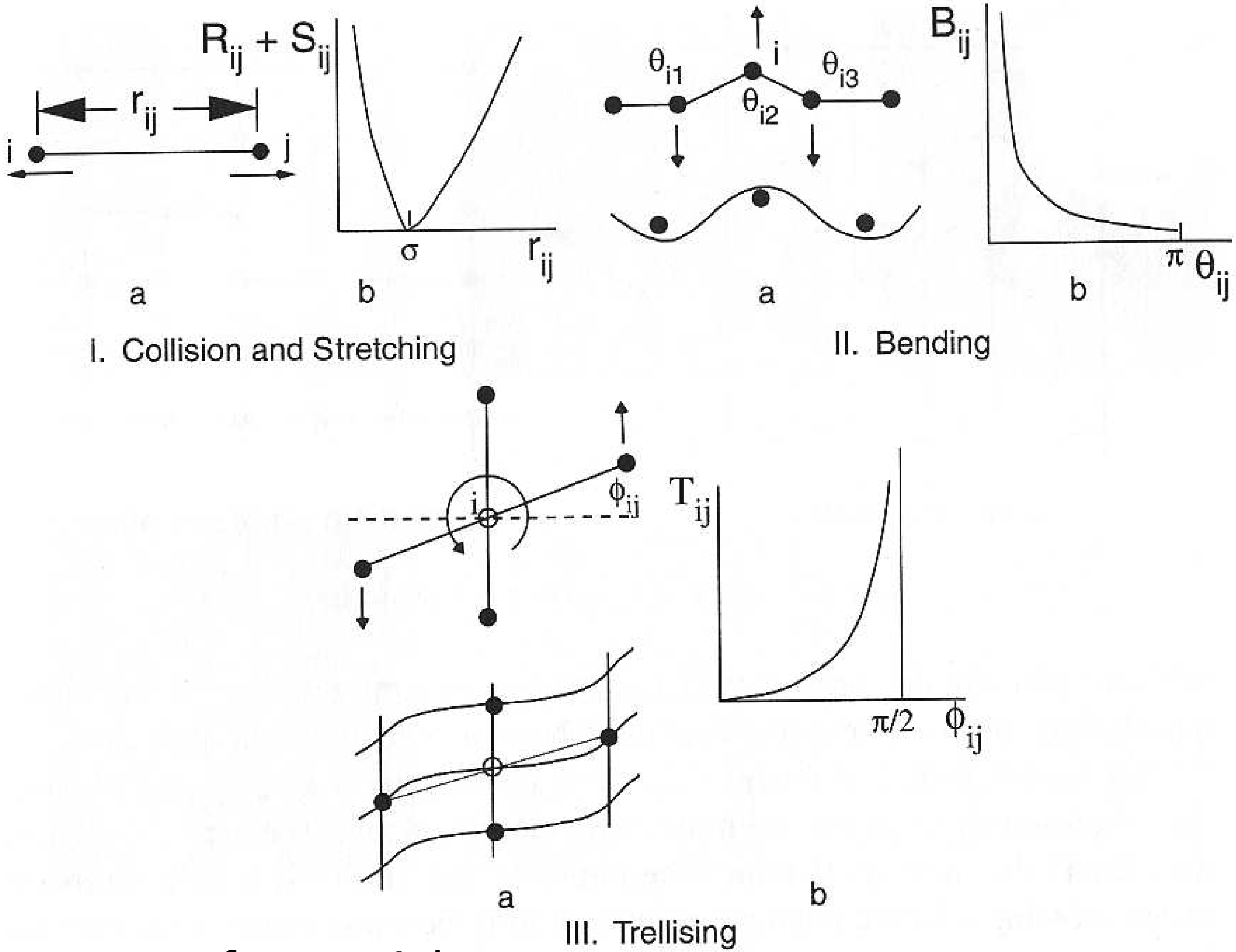


coton



wool

Modeling Drape (*after* [House, 2000])



the strain energy for particle i:

$$U_i = U_{repel_i} + U_{stretch i} + U_{bend i} + U_{trellis i}$$

Repulsion energy

- $U_{repel_i} = \sum_{j \neq i} R(r_{ij})$
- artificial energy – prevents self-intersection of the cloth
- for a single pair of particles i and j is given by function $R(r_{ij})$:

$$R(r_{ij}) = \begin{cases} C_0[(\sigma - r_{ij})^5 / r_{ij}] & r_{ij} \leq \sigma \\ 0 & r_{ij} > \sigma \end{cases}$$

- r_{ij} is distance of two particles, C_0 is a scale parameter and σ is a nominal distance where effects of repuslion and stretching changes (see fig. Ib)

Stretch energy

- $U_{stretchi} = \sum_{j \in N_i} S(r_{ij})$
- energy is expressed by the stretching function S
- incorporates the set of four-connected neighbor particles N_i (fig. Ib):

$$S(r_{ij}) = \begin{cases} 0 & r_{ij} \leq \sigma \\ C_0[((\sigma - r_{ij})/\sigma)^5] & r_{ij} > \sigma \end{cases}$$

Bend energy

- $U_{bendi} = \sum_{j \in M_i} B(\Theta_{ij})$
- bending energy depends on angle Θ determined by three neighbor particles (see fig. IIa)
- M_{ij} is the set of the nearest eight horizontal and vertical neighbors
- the bending energy should be at a minimum when the cloth is completely flat.
- it become large when the cloth is bended back on itself
- the bending energy is given as:

$$B(\Theta_{ij}) = C_1 \tan(\pi - \Theta_{ij})/2,$$

C_1 is a scale factor

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26

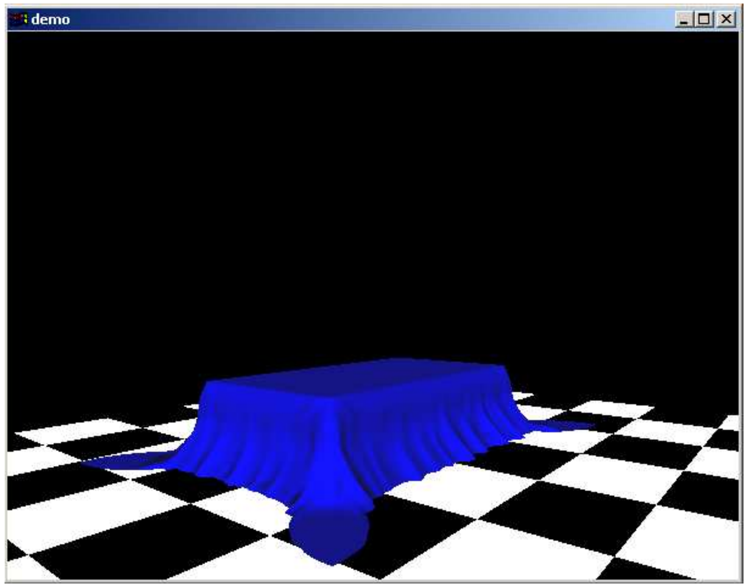
Trellising energy

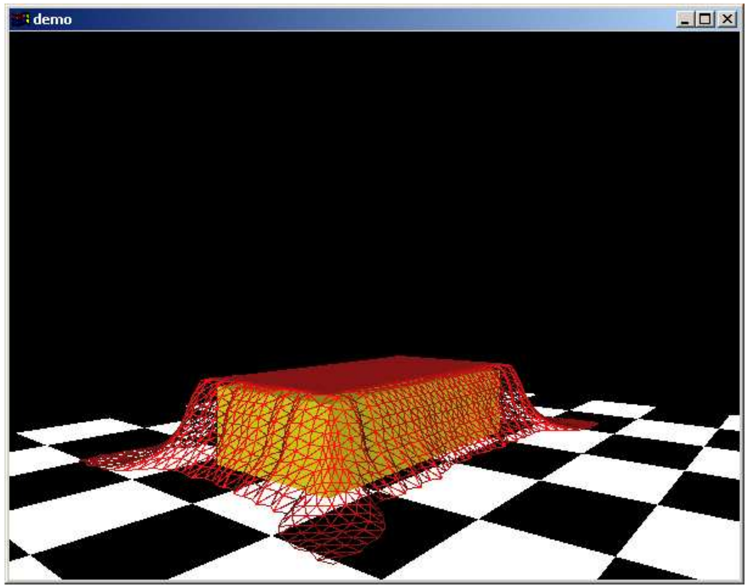
- $U_{trellisi} = \sum_{j \in K_i} T(\phi_{ij})$
- the energy is given by moving the line segments from equilibrium state (see fig. IIIa):

$$T(\phi_{ij}) = C_2 \tan(\phi_{ij})$$

- the angle ϕ is determined by this motion
- K_i is the set of four trellising angles formed around four connected neighbors of particle i
- the C_2 is scale parameter
- the trellising energy should be at minimum when crossing yarns are perpendicular

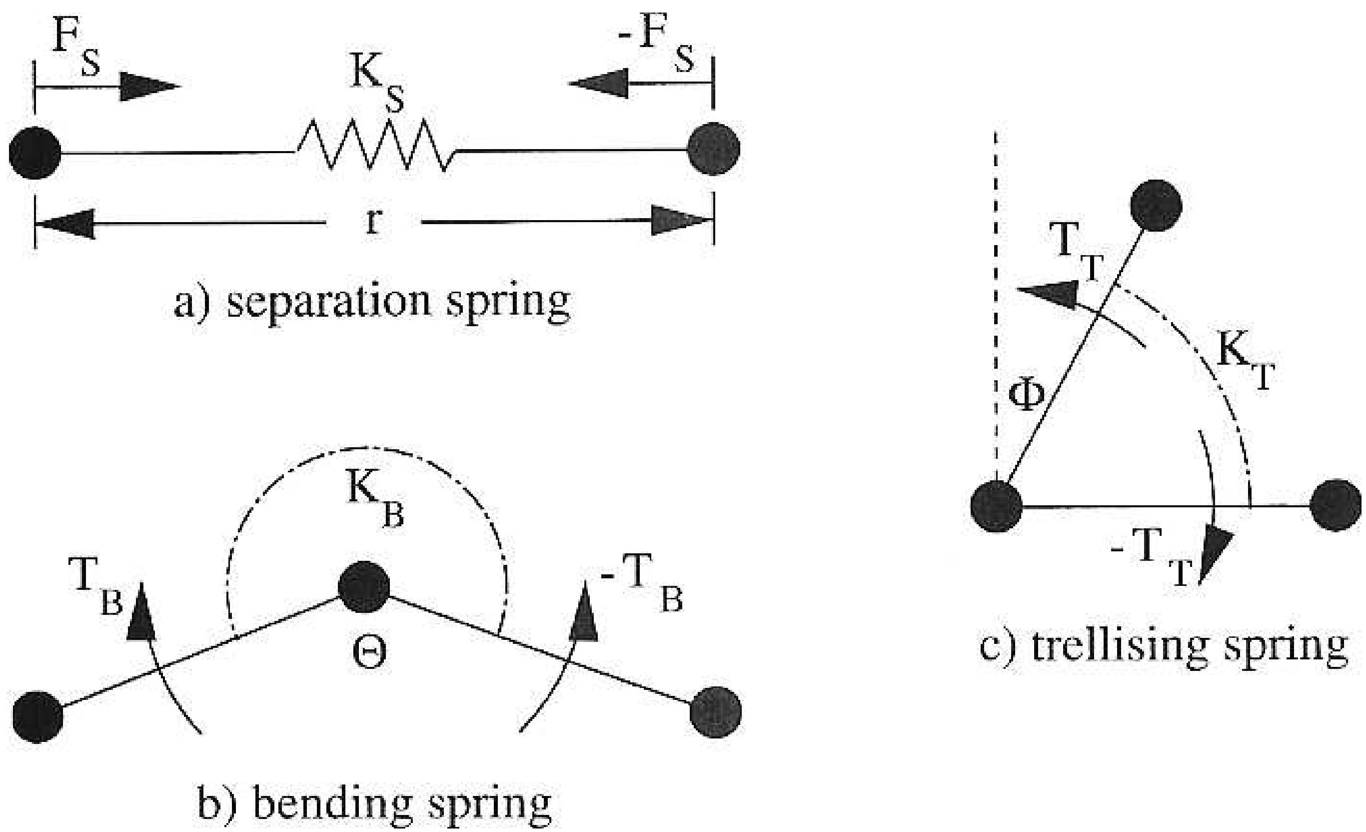
Modeling Drape





- the previous method using energy functions has two main disadvantages:
- it is computationally expensive
- it does not produce dynamic response for animation

Cloth Springs



- force-based formulation
- using non-linear "cloth springs"
- forces are obtained from derivations of appropriate energy functions

Separation Force

- acts to the two neighbor particles
- preserves the distance of the particle to be close to the nominal distance σ
- the separation spring K_S exerts force F_S:

$$F_S(r) = \frac{\partial(R+S)}{\partial r} = \begin{cases} -C_0(4r+\sigma)(r-\sigma)^4/\sigma^2 & r \leq \sigma \\ 5C_0(r-\sigma)^4/\sigma^5 & r > \sigma \end{cases}$$

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26

Bending Torque

- the torsional spring K_B exerts torque T_B opposing bending along a thread line
- T_B is given here as derivation of energy function from *Kawabata Evaluation System*: $T_B(\Theta) = \frac{978.8\sigma^2}{2}\left(\frac{\partial M}{\partial K} + M\right)\frac{\partial K}{\partial \Theta}$
- K is curvature and M is moment from Kawabata data
- based on empirical model, $\frac{\partial K}{\partial \Theta}$ is substituted: $T_B(\Theta) = -\frac{978.8\sigma^2}{2}\left(\frac{\partial M}{\partial K} + M\right)\sin\Theta/2$
- functions M and $\frac{\partial M}{\partial K}$ are obtained from the curve fit to the Kawabata bending data

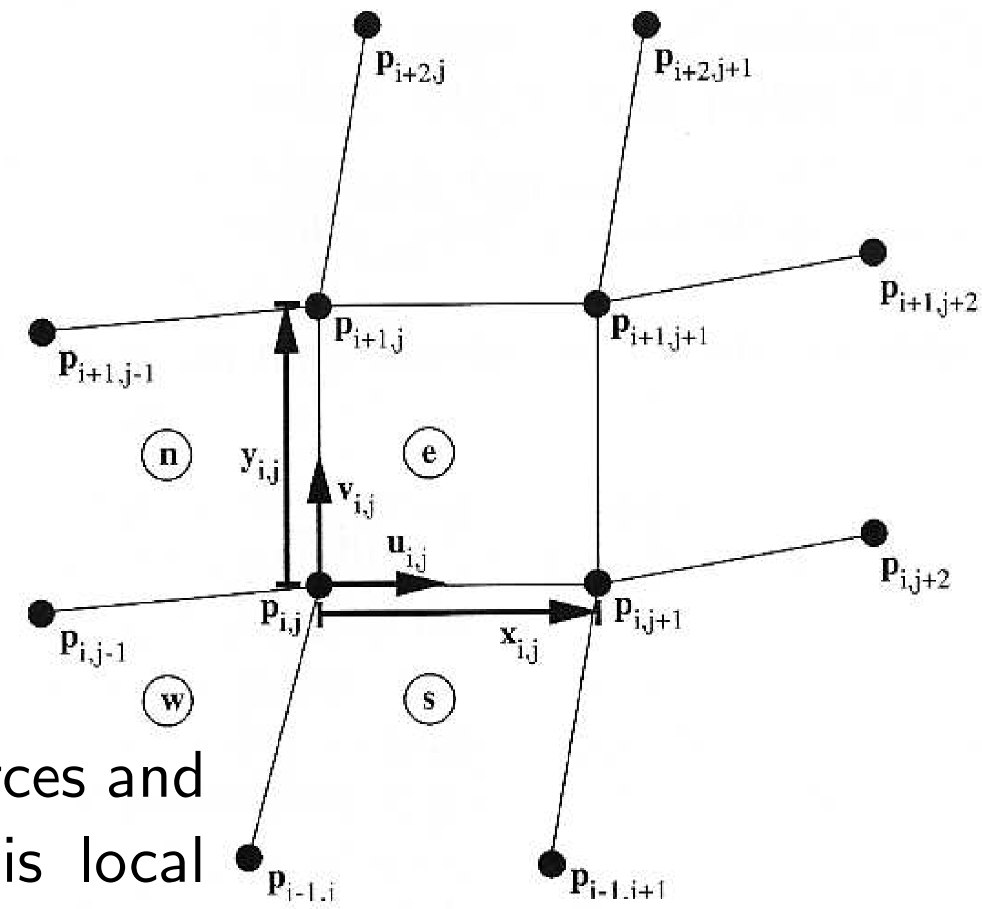
Trellising Torque

- the trellising torque is done by:

$$T_T(\phi) = \frac{195.76\sigma^3}{4} F \cos\phi$$

- F is obtained shearing force form the Kawabata data
- the sepparation force F_S and all torques are then incorporated in the cloth structure to construct motion equations

Final Model Structure



- the set of vectors expressing forces and torques are derivated from this local model [House, 2000]
- the mass of each particle is determined as mass of $\sigma \times \sigma$ area surrounding it
- $1/2$ of this mass for edge particle and $1/4$ for corner particle

Air Resistance

- can be determined at each particle using patch normals
- the force acting on each patch is determined by relative velocity:

$$\mathbf{U} = \mathbf{V} - \mathbf{W}$$

- the force $\mathbf{F}_A$ is then:

$$\mathbf{F}_A = C_3 \sigma^2 (\mathbf{n}.\mathbf{U})\mathbf{U}$$

- C_3 is a user-tunable constant related to the air density and surface characteristics of the cloth
- $\mathbf{n}$ is the surface normal vector

Conclusion

- the cloth modeling is wide area existing over 2 decades
- it is still subject of interest in real-time animation
- there are many methods extending the approaches presented
- uncovered topics omitted:
  - collision detection
  - rendering

References

[Weil,86] J. Weil The Synthesis of Cloth Objects *Proceedings of the Siggraph 1986*, volume 20(4), pages 49–53.

[Provot, 95] X. Provot, Deformation Contraints in a Mass-Spring Model to Describe Rigid Cloth Behavior *Proceedings of Graphics Interface, 1995*, pages 147–154.

[House, 2000] D. H. House and D. E. Breen Cloth Modeling and Animation *A K Peters, Ltd., 2000*,